

Matrices and Determinants

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \rightarrow ad - cb$$

$$\det \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \quad \begin{matrix} 2(2) - 3(1) \\ 4 - 3 \end{matrix}$$

$$\det \begin{bmatrix} 2x & 1 \\ 3 & 2 \end{bmatrix} = \begin{vmatrix} 2x & 1 \\ 3 & 2 \end{vmatrix} \quad \begin{matrix} 2x(2) - 3(1) \\ 4x - 3 \end{matrix}$$

$$\det \begin{bmatrix} -3 & -1 \\ -7 & -2 \end{bmatrix} = \begin{vmatrix} -3 & -1 \\ -7 & -2 \end{vmatrix} \quad \begin{matrix} -3(-2) - -7(-1) \\ 6 - 7 \\ -1 \end{matrix}$$

absolute value symbol does NOT mean absolute value, it means value, so answers can still be negative

Solving a system of equations with two variables - Cramer's Rule

Step 1: Find the coefficient matrix

Step 2: Substitute the constant terms for 'x' to solve for 'x'

Step 3: Substitute the constant terms for 'y' to solve for 'y'

$$\begin{matrix} \text{constant} \\ \underline{3x} - \underline{2y} = \underline{22} \\ \underline{x} + \underline{4y} = \underline{-2} \\ \text{constant} \end{matrix} \rightarrow \text{coefficient matrix} = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \xrightarrow{\text{determinant}} \begin{vmatrix} 3 & -2 \\ 1 & 4 \end{vmatrix} = 14$$

$$x = \frac{\begin{vmatrix} 22 & -2 \\ -2 & 4 \end{vmatrix}}{14} = \frac{84}{14} = 6$$

determinant of $\begin{bmatrix} 22 & -2 \\ -2 & 4 \end{bmatrix}$

Switch x's with constants

SOLUTION: (6, -2)

~~$$y = \frac{\begin{vmatrix} 3 & 22 \\ 1 & -2 \end{vmatrix}}{14} = \frac{-28}{14} = -2$$~~

Switch y's with constants

Determinant of a 3x3 Matrix

The Diagonal Method

12/9/10
Honors Algebra 2

Pd. A3

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} \begin{matrix} A_1 & B_1 \\ A_2 & B_2 \\ A_3 & B_3 \end{matrix}$$

Step 1: expand 1st two columns outside the brackets

Step 2: Draw diagonals to connect 3 entries per diagonal in the downward direction

$$A_1 B_2 C_3 + B_1 C_2 A_3 + C_1 A_2 B_3$$

Step 3: Repeat step 2 in the upward direction

$$A_3 B_2 C_1 + B_3 C_2 A_1 + C_3 A_2 B_1$$



$$A_1 B_2 C_3 + B_1 C_2 A_3 + C_1 A_2 B_3 -$$

$$(A_3 B_2 C_1 + B_3 C_2 A_1 + C_3 A_2 B_1)$$

$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ -2 & 2 & 3 \end{vmatrix} \begin{matrix} 1 & 3 \\ 2 & -1 \\ -2 & 2 \end{matrix}$$

$$\textcircled{1} 1(-1)(3) + 3(1)(-2) + -2(2)(2) -$$

$$\textcircled{2} -2(-1)(-2) + 2(1)(1) + 3(2)(3) \rightarrow 4 - 2 - 18 = 16$$

$$\textcircled{1} \boxed{-3 - 6 - 8} = -17 \neq -16 = \boxed{-33} \leftarrow \text{Determinant}$$

★ distribute negative

$$\begin{aligned}x + 2y - 3z &= -2 \\x - y + z &= -1 \\3x + 4y - 4z &= 4\end{aligned}$$

$$x = \frac{\begin{vmatrix} -2 & 2 & -3 \\ -1 & -1 & 1 \\ 4 & 4 & -4 \end{vmatrix}}{-7} = 0$$

$$\det \rightarrow \begin{vmatrix} x & y & z \\ 1 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 4 & -4 \end{vmatrix} = -7$$

$$y = \frac{\begin{vmatrix} 1 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 4 & 4 \end{vmatrix}}{-7} = 5$$

use graphing calculator to solve.